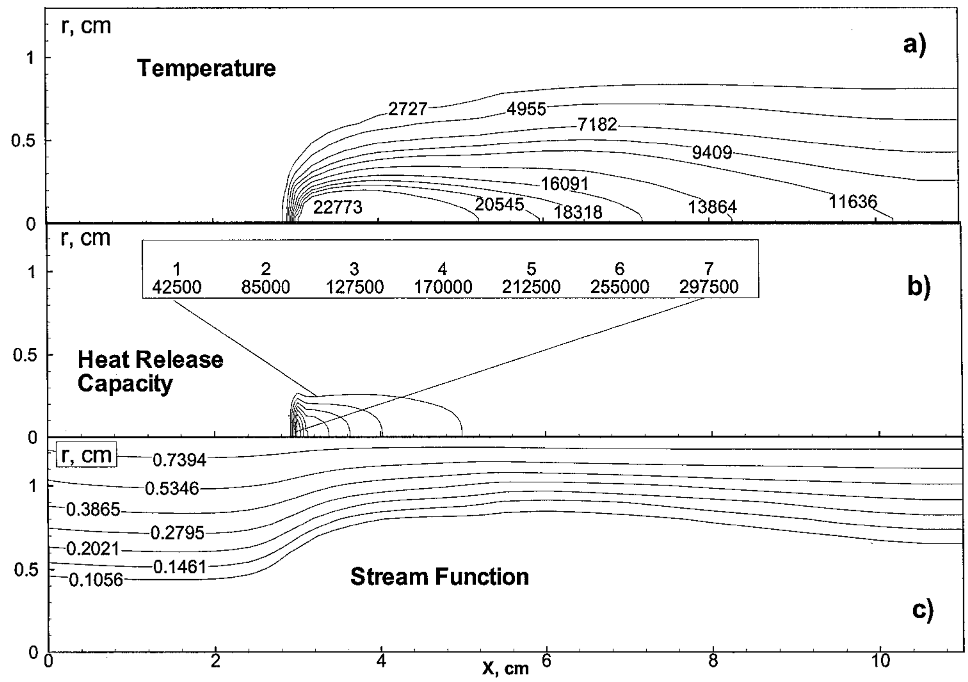


**Fig. 4 LPG gas dynamic parameters for $u_0 = 20$ m/s and $P_L = 100$ kW with turbulence entrance flow: a) temperature contours (numbers near the curves are the temperature in K), b) heat release capacity caused by laser radiation $Q_L$, W/cm$^3$, and c) stream function. Steady-state solution.**

auto-oscillations of the stream is less. For example, at $k = 0.01$ the influence of initial turbulence on thermal and gas dynamic functions is negligible.

## Conclusions

An unsteady radiative gas dynamic numerical simulation model of a laser plasma generator with turbulent gas stream is presented. This model is based on a system of coupled Navier–Stokes equations, laser radiation transfer and selective radiation heat-transfer equations, and also equations for the $k$-$\varepsilon$ model of turbulence.

It is determined that auto-oscillations of gas stream in the LPG do appear at certain critical input parameters. It was shown that a practical way to stabilize the gas dynamic structure in a LPG is by artificial turbulence of the input gas flow. The intrinsic gas-flow auto-oscillations in the LPG chamber may be suppressed by generation of as little as 10–20% amplitude oscillation for the input gas velocity.

## Acknowledgments

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# Statistical Heat Transfer from Uniform Annular Fins with High Thermal Conductivity Coating

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## Nomenclature

| | | |
|---|---|---|
| $Bi$ | = | Biot number, $ht/k$ |
| $c$ | = | inner-to-outer radii ratio, $r_1/r_2$ |
| $h$ | = | convection coefficient, W/m$^2\cdot$K |
| $I_\nu(\cdot)$ | = | modified Bessel function of first kind and order $\nu$ |
| $K_\nu(\cdot)$ | = | modified Bessel function of second kind and order $\nu$ |
| $k_1$ | = | thermal conductivity of material 1, W/m$\cdot$K |
| $k_2$ | = | thermal conductivity of material 2, W/m$\cdot$K |
| $\bar{k}$ | = | generic spatial mean of the thermal conductivities $k_1$ and $k_2$, W/m$\cdot$K |
| $L$ | = | length, $r_2 - r_1$, m |
| $L_t$ | = | total length, $r_2 - r_1 + t_2$, m |
| $Q$ | = | heat transfer rate, W |
| $Q_i$ | = | ideal heat transfer rate, W |
| $R$ | = | normalized dimensionless $r$, $r/r_2$ |
| $r$ | = | radial variable, m |
| $r_1$ | = | inner radius, m |
| $r_2$ | = | outer radius, m |
| $T$ | = | temperature, K |
| $T_b$ | = | base temperature, K |
| $T_\infty$ | = | fluid temperature, K |
| $t$ | = | total semithickness, $t_1 + t_2$, m |

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$t_1$ = semithickness of material 1, m
$t_2$ = semithickness of material 2, m
$\bar{\beta}^2$ = spatial mean thermogeometric parameter, $h/\bar{k}t$, m$^{-2}$
$\bar{\gamma}^2$ = dimensionless $\bar{\beta}^2$, $\bar{\beta}^2 r_2^2$
$\eta$ = fin efficiency or dimensionless $Q$, $Q/Q_i$
$\theta$ = normalized dimensionless $T$, $(T - T_\infty)/(T_b - T_\infty)$
$\phi$ = correction factor in Eq. (15)

*Subscripts*

$a$ = arithmetic
$g$ = geometric
$h$ = harmonic

## Introduction

CONVECTIVE heat transfer from a solid surface to a surrounding fluid may be increased by attaching fins to the solid surface. For example, some surfaces of heat exchanger tubes are finned, especially on the gas side where the convection coefficient is low. A large variety of fin configurations are manufactured for heat exchanger tubes, and the most popular geometries used in industry are documented by Webb.[1] Arguably, the main challenge faced by design engineers is to find ways, if possible, to devise an aggregate effect from a commensurate augmentation of heat transfer from a finned base surface in contact with a gas.

A coated annular fin is defined as an annular fin made from a primary material (substrate) that is coated with a layer of a high conducting material (coating) to further enhance the transfer of heat. This novel type of annular fin is encountered in a special class of high-performance heat exchangers, whose finned tubes are constructed with carbon steel as the primary material and are coated with zinc. To impregnate the layer of zinc, the final phase of the manufacturing process of these finned tubes requires their immersion in a bath of liquid zinc (see Lindsay[2]).

The thermal analysis of coated annular fins is a complicated matter, and only one article by Lalot et al.[3] is available in the literature so far. In Ref. 3, the authors formulated a two-dimensional conjugate model with a pair of coupled elliptic partial differential heat conduction equations in the two domains: the substrate and the coating. The authors obtained an intricate analytical solution for the two temperature fields in terms of Bessel–Fourier infinite series that led subsequently to an elaborate analytical expression for the fin efficiency. In general, it was claimed in Ref. 3 that the efficiency of regular annular fins may be considerably increased by the addition of a coating of thermal conductivity higher than the thermal conductivity of the substrate. Also, the authors detected that the efficiency of coated annular fins with a moderate radius difference may raise by a factor of almost two when compared with the standard uncoated annular fins of the same size.

This Technical Note addresses a largely simplified thermal analysis of coated annular fins that seeks to avoid the innate two-dimensional conjugate model adopted in Ref. 3. The simplified thermal analysis to be developed here engages a statistical variant of the quasi-one-dimensional classical model for standard uncoated annular fins made from one material (see Harper and Brown[4] and Schmidt[5]). Basically, this fundamental model relies on an ordinary differential equation, the modified Bessel equation of zero order, but encompassing suitable spatial means of the thermal conductivities of the substrate and the coating. Within the framework of statistical theory (see Hardy et al.[6]), the three spatial means that seem to be pertinent to the present model are 1) the arithmetic spatial mean, 2) the geometric spatial mean, and 3) the harmonic spatial mean, respectively.

## Variant of a Quasi-One-Dimensional Model

The annular fin assembly shown in Fig. 1 is made from a primary material (substrate) with uniform semithickness $t_1$ and thermal conductivity $k_1$. The inner and outer radius of the annular fin are $r_1$ and $r_2$, respectively. To intensify the heat transfer further, the annular fin is coated with a very thin layer of a material of higher thermal conductivity $k_2(\gg k_1)$ and semithickness $t_2(\ll t_1)$. As compared to

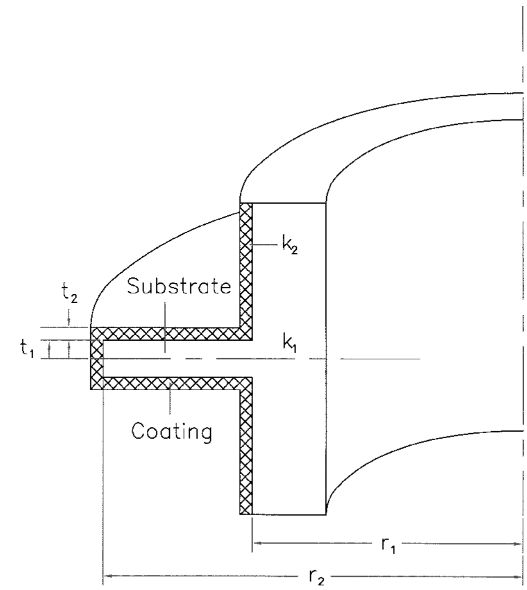


**Fig. 1 Coated annular fin of uniform thickness.**

the simple annular fin, this new geometric arrangement creates a composite annular fin that has enlarged dimensions, that is, a total semithickness $t = t_1 + t_2$ and a total length $L = r_2 - r_1 + t_2$. The model is the quasi-one-dimensional heat conduction equation for constant properties:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{h}{\bar{k}t}(T - T_\infty) = 0 \qquad \text{in } r_1 \le r \le r_2 \tag{1}$$

where $\bar{k}$ is a generic spatial mean of the thermal conductivities of the two intervening materials, namely, $k_1$ for the substrate and $k_2$ for the coating. In actuality, this alternate approach is synonymous with the replacement of a two-material annular fin by a conventional annular fin made from a homogeneous single material; the latter possesses a spatial-mean thermal conductivity $\bar{k}$ that encapsulates $k_1$ and $k_2$. Equation (1) rests on the assumption that the coated annular fin is sufficiently thin, so that the temperature gradient occurs in the radial direction predominantly; this issue results in a transversal Biot number, $\mathrm{Bi} < 0.1$ (see Lau and Tan[7]). When a constant temperature at the base and an adiabatic tip are specified, the respective boundary conditions are,

$$T = T_b, \qquad r = r_1 \tag{2a}$$

and

$$\frac{dT}{dr} = 0, \qquad r = r_2 \tag{2b}$$

The heat transfer from a coated annular fin to a fluid can be computed by Fourier's law at the base

$$Q = -\bar{k}(2\pi r_1)(2t)\left.\frac{dT}{dr}\right|_{r=r_1} \tag{3}$$

where, as before, $\bar{k}$ is a generic spatial-mean thermal conductivity.

Statistical theory[6] supplies an assortment of means and spatial means for discrete data. Among the set of spatial means available, the subset that is apt for the estimation of the equivalent thermal conductivities $\bar{k}$ may be defined as follows:

The arithmetic spatial mean:

$$\bar{k}_a = \frac{k_1 t_1 + k_2 t_2}{t_1 + t_2} \tag{4}$$

The geometric spatial mean:

$$\bar{k}_g = \left(k_1^{t_1} k_2^{t_2}\right)^{1/(t_1 + t_2)} \tag{5}$$

The harmonic spatial mean:

$$\bar{k}_h = 1/\{[1/(t_1 + t_2)](t_1/k_1 + t_2/k_2)\} \tag{6}$$

Further, it has been demonstrated in Ref. 6 that the preceding three spatial-mean thermal conductivities are quantitatively related by the sequence of inequalities

$$\bar{k}_a > \bar{k}_g > \bar{k}_h \tag{7}$$

Introduction of the normalized dimensionless variables $\theta$ for the temperature and $R$ for the radial variable

$$\theta = (T - T_\infty)/(T_b - T_\infty), \qquad R = r/r_2 \tag{8}$$

transforms Eqs. (1) and (2) into

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}\theta}{\mathrm{d}R} - \bar{\gamma}^2\theta = 0 \qquad \text{in } c \le R \le 1 \tag{9}$$

together with the boundary conditions

$$\theta = 1, \qquad R = c \tag{10a}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}R} = 0, \qquad R = 1 \tag{10b}$$

Clearly, the dimensionless thermogeometric parameter $\bar{\gamma}^2 = \bar{\beta}^2 r_2^2$ and the inner-to-outer radii ratio $c = r_1/r_2$ appearing in Eqs. (9) and (10a) emerge spontaneously during the course of the nondimensionalization of Eqs. (1) and (2).

The exact analytical solution of Eq. (9) subject to Eq. (10), taken directly from Jakob,[8] yields the temperature distribution along a uniform annular fin:

$$\theta(R) = \frac{K_1(\bar{\gamma})I_0(\bar{\gamma}R) + I_1(\bar{\gamma})K_0(\bar{\gamma}R)}{K_1(\bar{\gamma})I_0(\bar{\gamma}c) + I_1(\bar{\gamma})K_0(\bar{\gamma}c)} \tag{11}$$

It is common practice to calculate the heat transfer $Q$ from fins, by way of the fin efficiency or dimensionless heat transfer $\eta = Q/Q_i$ proposed by Gardner.[9] In this ratio, $Q_i$ designates the heat transfer from an ideal fin, which is an identical fin made from a hypothetical material with infinite thermal conductivity that maintains an even temperature equal to its base temperature $T_b$. Thus, from Eqs. (3) and (8), the corresponding expression for the efficiency of a uniform annular fin becomes

$$\eta = \frac{-2c(\mathrm{d}\theta/\mathrm{d}R)|_{R=c}}{\bar{\gamma}^2(1 - c^2)} \tag{12}$$

Then, substituting Eq. (11) into Eq. (12) results in the lengthy relation

$$\eta = \frac{-2c}{\bar{\gamma}(1 - c^2)}\left[\frac{K_1(\bar{\gamma})I_1(\bar{\gamma}c) - I_1(\bar{\gamma})K_1(\bar{\gamma}c)}{K_1(\bar{\gamma})I_0(\bar{\gamma}c) + I_1(\bar{\gamma})K_0(\bar{\gamma}c)}\right] \tag{13}$$

which obviously depends on $\bar{\gamma}$ and $c$ only. Conversely, from a practical standpoint, it may be advantageous to replace the complex form of Eq. (13) containing four modified Bessel functions with a facile equation including a single hyperbolic tangent, sort of resembling the efficiency of the limiting case of uniform straight fins ($c = 1$). In this regard, Schmidt[10] found the accurate relation

$$\eta = \tanh(mL\phi)/mL\phi \tag{14}$$

where $(mL)^2 = (h/\bar{k}t)(r_2 - r_1)^2$ is the dimensionless thermogeometric parameter that is typical of a uniform straight fin. The correction factor $\phi$ in Eq. (14) is given by

$$\phi = 1 + 0.35\,\ln(r_2/r_1) \tag{15}$$

Worthy thermal designs of uniform annular fins for heat exchanger tubes are characterized by moderate-to-high fin efficiencies of the order of $\eta > 0.5$ (Ref. 1). For these effective finned tubes, the deviations between the approximate and the exact predictions of $\eta$ with Eqs. (13) and (14) do not exceed 1%.

Inasmuch as $\bar{k}_a$, $\bar{k}_g$, and $\bar{k}_h$ constitute the formal statistical bounds for the spatial-mean thermal conductivities of the substrate and the coating,[6] evaluation of Eqs. (11) and (13) with embedded values of the spatial-mean thermogeometric parameters $\bar{\beta}_a^2$, $\bar{\beta}_g^2$, and $\bar{\beta}_h^2$ equally furnish statistical bounds for the estimations of the temperature distribution and the companion efficiency of coated annular fins.

## Statistically Bounded Fin Efficiencies

In Ref. 3, the two bidimensional temperature fields computed analytically, one for the substrate and the other for the coating, were channeled through an expression for a two-dimensional thermogeometric parameter,

$$\beta_{2\text{-D}}^2 = h/(k_1t_1 + k_2t_2) \tag{16}$$

It is now convenient to examine Eq. (16) from a different perspective. This step may be done with the help of the defining expressions for the spatial-mean thermal conductivities of the substrate, $k_1$, and of the coating, $k_2$, supplied by Eqs. (4–6). Focusing our attention momentarily on the arithmetic spatial mean $\bar{k}_a$ in Eq. (4) enables us to replace the denominator $k_1t_1 + k_2t_2$ in Eq. (16) with the product $\bar{k}_a(t_1 + t_2)$. Because the total semithickness $t = t_1 + t_2$, it is easy to recognize the equality $\bar{k}_a(t_1 + t_2) = \bar{k}_a t$, and ultimately Eq. (16) gets reduced to

$$\beta_{2\text{-D}}^2 = h/\bar{k}_a t \tag{17}$$

In view of the foregoing, the stage is now set to assess the goodness of the simplistic quasi-one-dimensional model with appended spatial-mean thermal conductivities of the substrate and the coating [Eqs. (4–6)] relative to the formal two-dimensional model.[3] Reference 3 presents a real problem that is connected to the design of high-performance heat exchanger tubes and evidently comprises a wide range of real design parameters commonly employed in industry. The spectrum of the design parameters is described next. The tubes are circumscribed with an array of coated annular fins made of carbon steel ($k_1 = 50$ W/m · K) and coated by zinc ($k_2 = 111$ W/m · K). The tube diameter $2r_1$ is usually constrained between 10 and 50 mm, and the outer diameter of the annular fins $2r_2$ is 1.5, up to three times the size of the tube diameter. The semithickness of the substrate takes values from $t_1 = 0.1$ to 0.5 mm and the semithickness of the coating $t_2$ normally ranges between 25 and 75 $\mu$m. The convection coefficient customarily varies from 25 to 150 W/m$^2$ · K. Note that the working fluids utilized in heat exchanger finned tubes cannot surpass the melting point of the coating, for instance, the melting point of zinc and zinc oxide are 420°C and 1975°C, respectively.

The central goal of the parametric study in Ref. 3 was to compare the fin efficiencies of advanced coated annular fins against the fin efficiencies of ordinary uncoated annular fins for various combinations of the influencing design parameters proper to heat exchanger environments. The main conclusion drawn in Ref. 3 was that the efficiency of a coated annular fin gets magnified as a response to gradual increments in the thickness of the coating. A logical figure of merit to judge the performance levels achieved by the coated annular fins seems to be the fin efficiency ratio (FER) $= \eta_{\text{coated}}/\eta_{\text{uncoated}}$. In this sense, among the margins of augmentation found in Ref. 3 for FER, the largest margin was of the order of 1.9, that is, nearly a double efficiency. This beneficial feature was produced with the design of a fin–tube assembly having dimensions $r_1 = 10$ mm, $r_2 = 30$ mm, $t_1 = 0.1$ mm, and $t_2 = 150$ $\mu$m, coupled with a fair convection coefficient $h = 50$ W/m$^2$ · K. The final result of the comparison is shown in the upper right corner of Fig. 5 in Ref. 3.

Using the same data that rendered the largest FER (=1.9) in Ref. 3, the three spatial-mean thermal conductivities may be obtained

**Table 1 Comparison of the actual fin efficiencies of uncoated and coated annular fins that grant the largest FER = 1.9 in Ref. 3**

| Material | $\eta$ Two-dimensional model | $\eta$ Quasi-one-dimensional model | $\eta_a$ Quasi-one-dimensional model | $\eta_g$ Quasi-one-dimensional model |
|---|---|---|---|---|
| Carbon steel | 0.34 | 0.34 | —— | —— |
| Carbon steel/zinc | 0.66 | —— | 0.66 | 0.64 |

immediately for the convertibility of the coated annular fin: 1) the arithmetic spatial mean, $\bar{k}_a = 86.6$ W/m · K, 2) the geometric spatial mean, $\bar{k}_g = 80.68$ W/m · K, and 3) the harmonic spatial mean, $\bar{k}_h = 74.6$ W/m · K, respectively. Notice that the descending sizes of these numbers are in conformity with the sequence of inequalities stipulated in Eq. (7). Also, it is curious to observe the behavior of the three spatial-mean thermal conductivities. First, the largest $\bar{k}_a = 86.6$ W/m · K and the smallest $\bar{k}_h = 74.6$ W/m · K are 60% and 49% higher than the thermal conductivity of the substrate $k_1$ made from carbon steel, respectively. Second, the middle $\bar{k}_g$ is equidistant between the two extreme spatial-mean thermal conductivities, $\bar{k}_a$ and $\bar{k}_h$; each separation is roughly equal to $\Delta\bar{k} = 6$ W/m · K.

Undoubtedly, the ultimate measure of the performance of the approximate quasi-one-dimensional model developed here vs the exact two-dimensional model elaborated in Ref. 3 is the computation of the fin efficiency or dimensionless heat transfer. The computed values of the efficiencies by way of the two contrasting models are listed in Table 1. It may be confirmed in Table 1 that the approximate estimate of the one-dimensional efficiency $\eta_a = 0.66$ based on the arithmetic spatial mean of the thermal conductivities $\bar{k}_a = 86.6$ W/m · K coincides perfectly with the exact two-dimensional efficiency of 0.66 of Ref. 3. Moreover, the approximate estimate of the one-dimensional efficiency of 0.64 based on the geometric spatial mean of the thermal conductivities $\bar{k}_g = 80.68$ W/m · K lies slightly below the exact two-dimensional efficiency of 0.66 of Ref. 3 and exhibits a diminute separation of $\Delta\eta = 0.02$ units only. Certainly, it is evident that $\eta_{\bar{k}_a} \approx \eta_{\bar{k}_g}$. This agreement is extremely favorable because it shrinks the band of error for $\eta$, bringing the error to almost zero. Besides, it may be inferred that the remaining combinations of design parameters tested in Ref. 3 may be handled equally well with the statistical procedure developed in this Technical Note. The comparisons are not included for brevity.

In theory, the real value of the efficiency $\eta$ of a coated annular fin must be confined to the ample interval $\eta_a \geq \eta \geq \eta_g$, which is imposed by statistical theory[6]; $\eta_a$ is an upper bound and $\eta_g$ is a lower bound. However, in engineering practice, the outcome of this Technical Note has categorically demonstrated that the first bound, $\eta_a$, is a strong upper bound, which overlaps with the exact $\eta$ in Ref. 3 and is associated with the maximum heat removal from the coated annular fin. In addition, the second bound, $\eta_g$, is connected to the minimal heat rejection from the coated annular fin and gives a strong lower bound. Therefore, the third bound, $\eta_h$, being a weak lower bound, may be discarded for practical purposes simply because it has no bearing on the thermal performance of the coated annular fins under study.

Attention is now turned to the physical interpretation of the statistical results for the efficiency found. It may be said that the rightness of the arithmetic spatial mean of the thermal conductivities $\bar{k}_a$ in Eq. (4) is in harmony with the series arrangement of the two-layered materials (the substrate and the coating) in the uniform annular fin in Fig. 1. In the same vein, the harmonic spatial mean of the thermal conductivities $\bar{k}_h$ in Eq. (6) is defective because of its association with the parallel arrangement of two-layered materials, which is not the case in Fig. 1.

Knowledge of the optimum dimensions of fins has become an important requirement in contemporary fin design, and for completeness some comments about this topic are in order. The optimum dimensions of standard uncoated annular fins of uniform thickness are those dimensions that give the greatest amount of heat transfer $Q_{\max}$ for a given mass of material (fixed $t$, $r_1$, and $r_2$). This information was presented for the first time by Jakob[8] in the form of an elementary nomogram and later by Brown[11] and Ullmann and Kalman[12] in refined graphical formats. Naturally, the optimization schemes created in Refs. 11 and 12 for bare annular fins apply equally well for coated annular fins, hence supplying strong upper and strong lower bounds for the fin dimensions. As in the case of the temperature distribution and the efficiency, the thermal conductivity for a single material, $k$, needs to be replaced by the expanded arithmetic and geometric spatial means of the thermal conductivities of the substrate and the coating, that is, $\bar{k}_a$ and $\bar{k}_g$, respectively.

## Conclusions

Coated annular fins made from a substrate (primary material) covered with a coating (secondary material) of high thermal conductivity are frequently affixed to tubes of high-performance heat exchangers. The simplistic quasi-one-dimensional model with embedded statistical spatial means of the thermal conductivities of the substrate and the coating may permit design engineers to estimate quickly upper and lower bounds for the temperature distributions and efficiencies of this kind of unconventional fin. The upper bounds are provided by the arithmetic spatial mean of the thermal conductivities $\bar{k}_a$, from Eq. (4), whereas the lower bounds are furnished by the geometric spatial mean of the thermal conductivities $\bar{k}_g$, from Eq. (5). The prevalent statistical concept may be particularized to coated straight fins attached to plain walls.

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